6.S062: Mobile and Sensor Computing

Lecture 6: Introduction to Inertial Sensing & Sensor Fusion

Some material adapted from Gordon Wetzstein (Stanford) and Sam Madden (MIT)

Example Application: Inertial Navigation

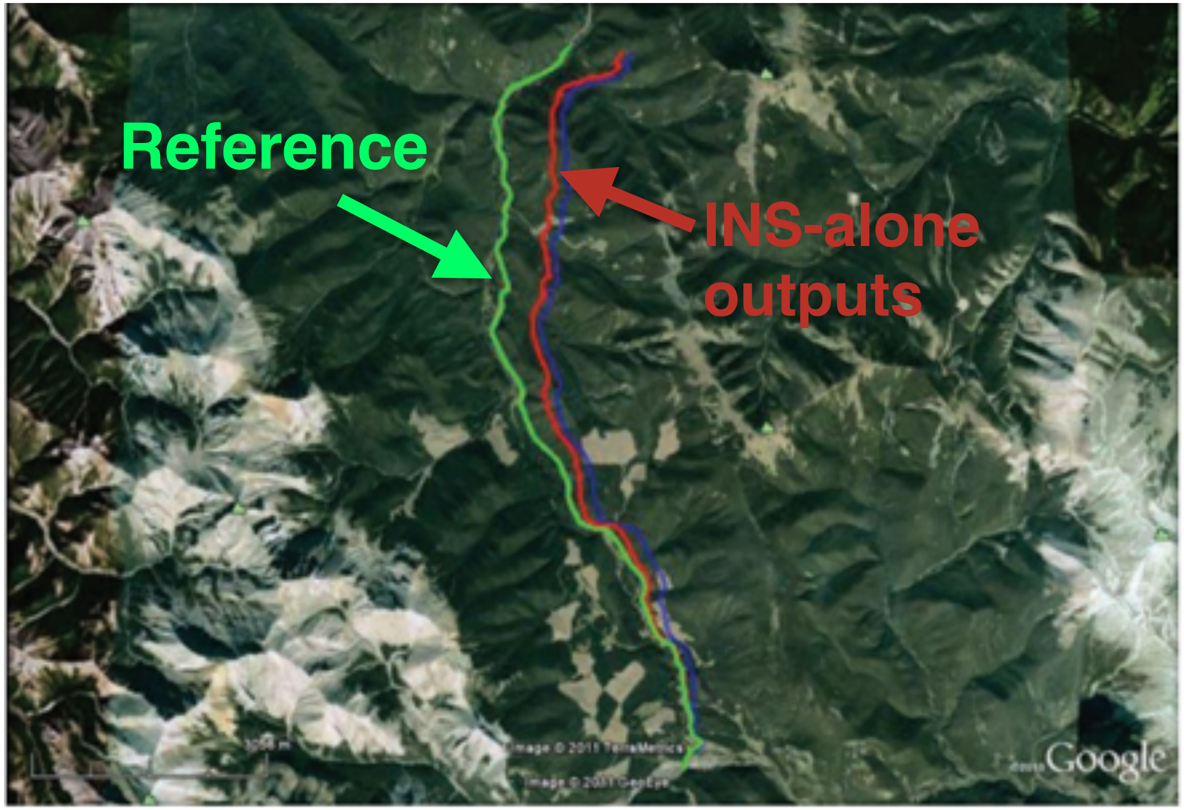
GPS only

GPS+INS

<u>Key Idea #1:</u> Integrate acceleration data over time to discover location (Inertial Sensing)

Inertial Sensing alone is not enough for accurate positioning

- Errors accumulate over time



Source: INS Face Off MEMS versus FOGs

<u>Key Idea #2:</u> Fuse Data from Multiple Sensors (Sensor Fusion)

This Lecture

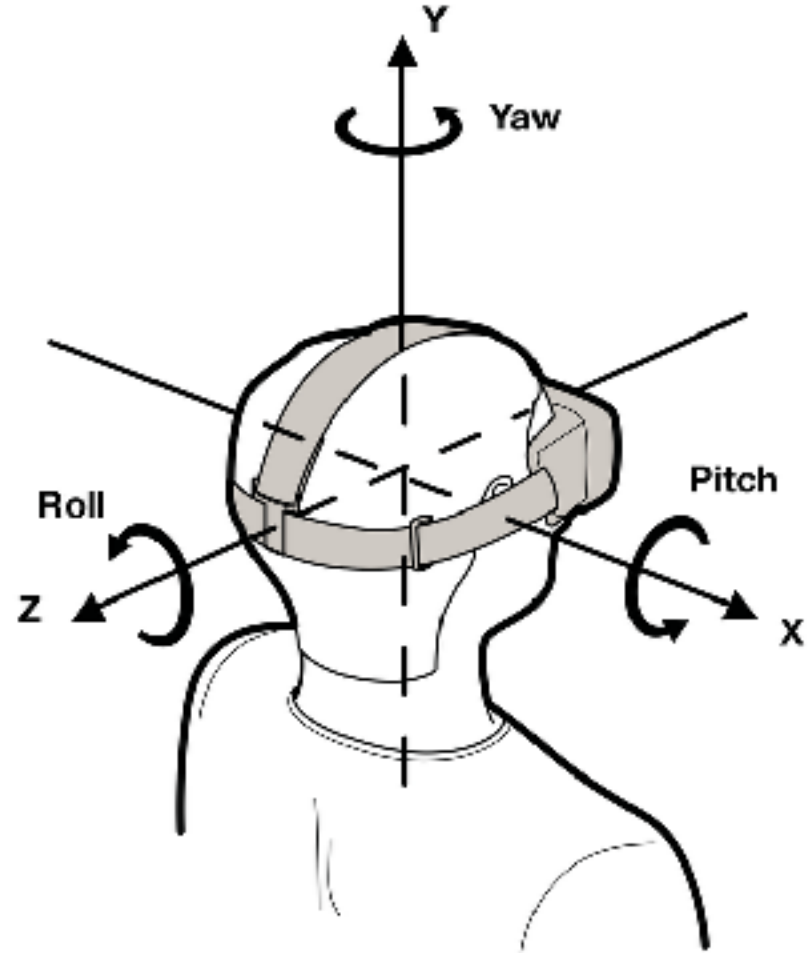
Key Idea #1: Integrate acceleration data over time to discover location (Inertial Sensing)

Key Idea #2: Fuse Data from Multiple Sensors (Sensor Fusion)

Let’s understand inertial sensing in the context of VR

- **Goal:** track location and orientation of head or other device

- **Coordinates:** Six degrees of freedom:
  - Cartesian frame of reference (x,y,z)
  - Rotations represented by Euler angles (yaw, pitch roll)



Source: Occulus

What does an IMU consist of? (Inertial Measurement Unit)

- **Gyroscope** measures angular velocity **ω** in degrees/s
- **Accelerometer** measures linear acceleration **a** in m/s^2
- **Magnetometer** measures magnetic field strength **m** in μT (micro-Teslas).

Why is it called IMU?

History of IMUs

- Earliest use of gyroscopes goes back to German ballistic missiles (V-2 rocket) in WW2 for stability)

- In the 1950s, MIT played a central role in the development of IMUs (Instrumentation Lab)

Uses of IMUs

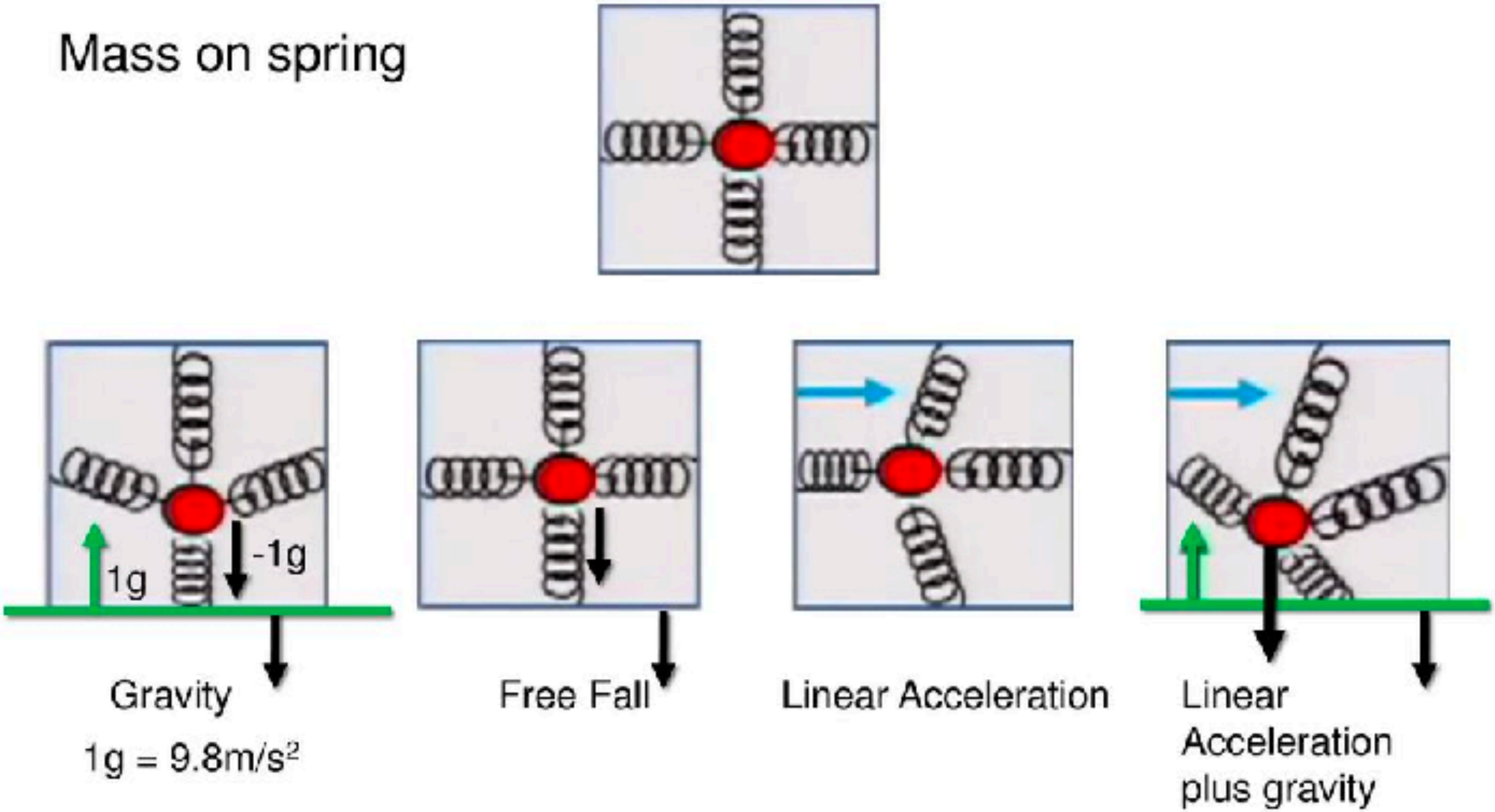
Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer

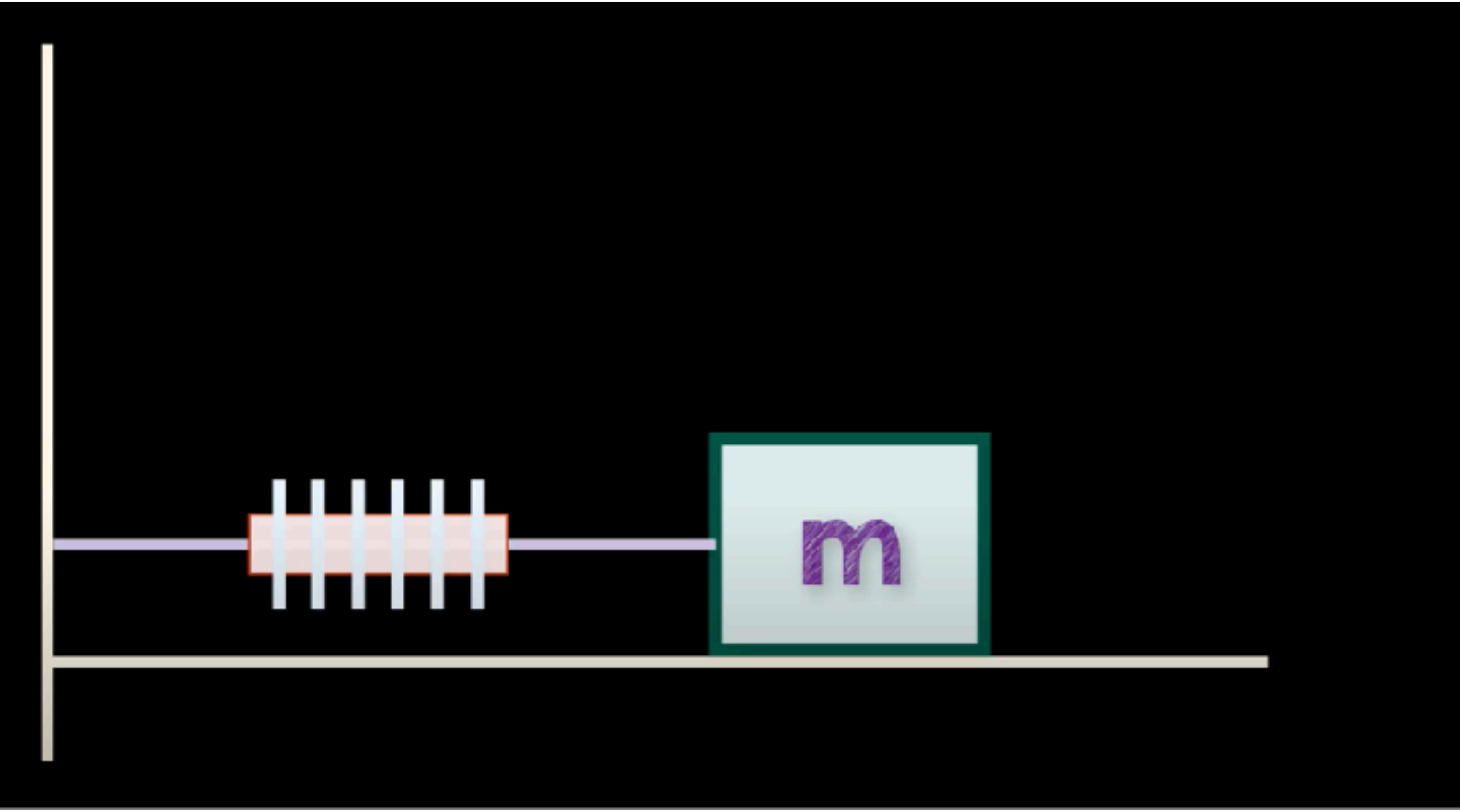
- Understanding Sources of Errors

- Dead reckoning by fusing multiple sensors

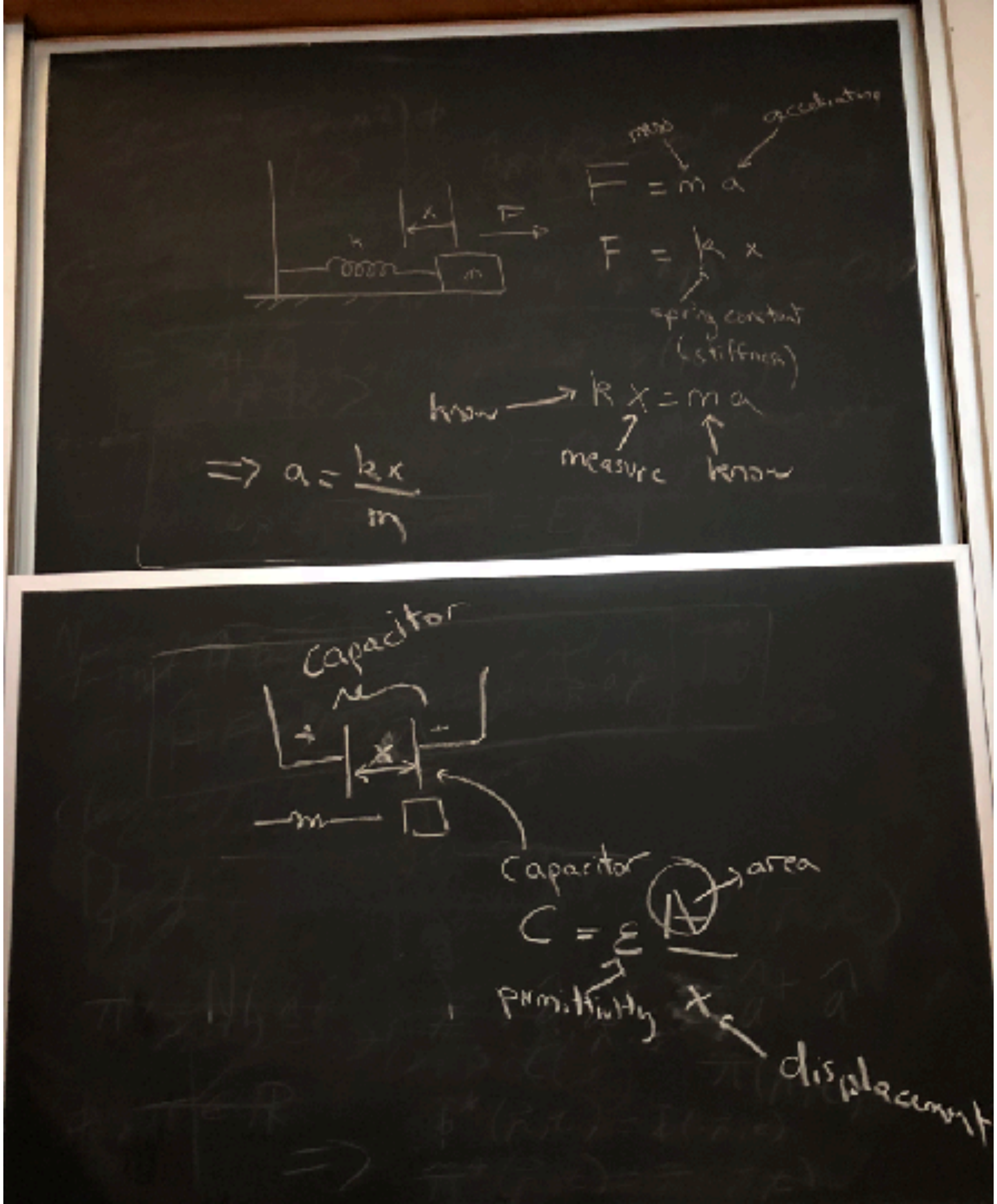
How Accelerometers Work



How Accelerometers Work

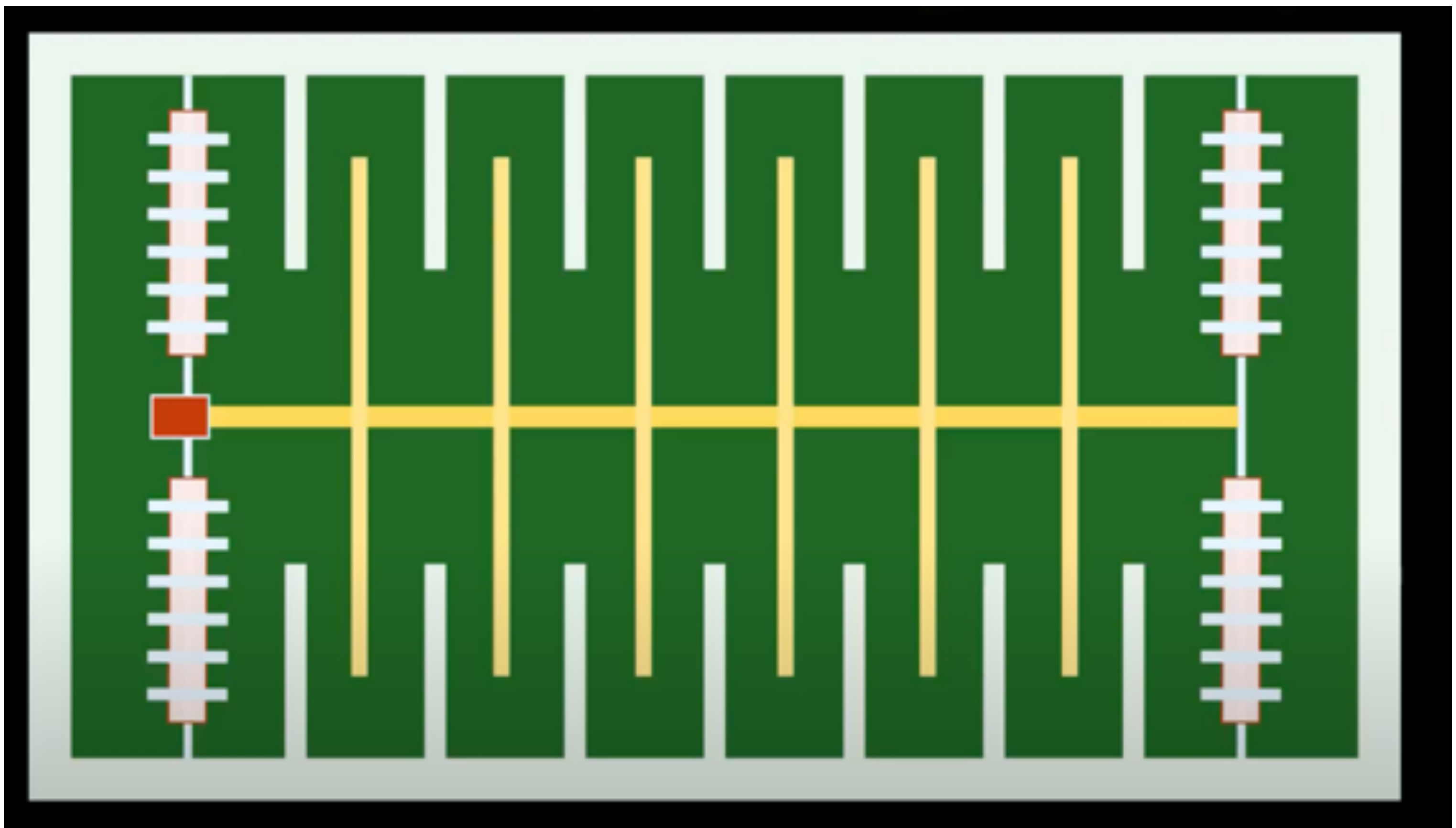


What matters is the displacement


mass
acceleration
F = m a
F
x
k
m
F = k x
spring constant
(stiffness)
know → k x = m a
measure
know
⇒ a = kx / m
Capacitor
+
−
x
m
capacitor
area
C = ε A / x
permittivity
displacement

Measuring Displacement

- How do we measure displacement?
- Most common approach is to use capacitance and MEMS (Micro electro-mechanical systems)



Measuring Displacement

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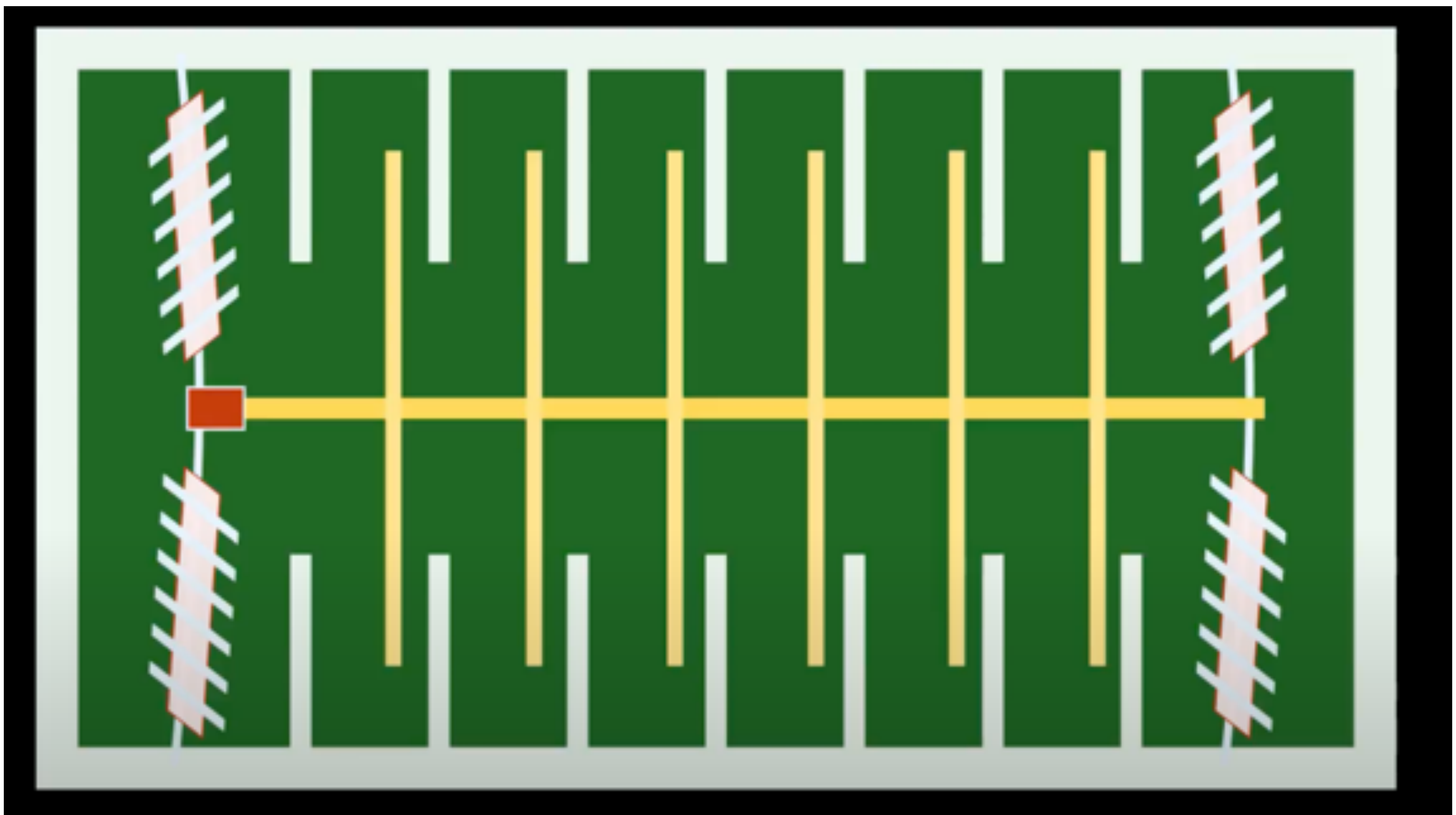
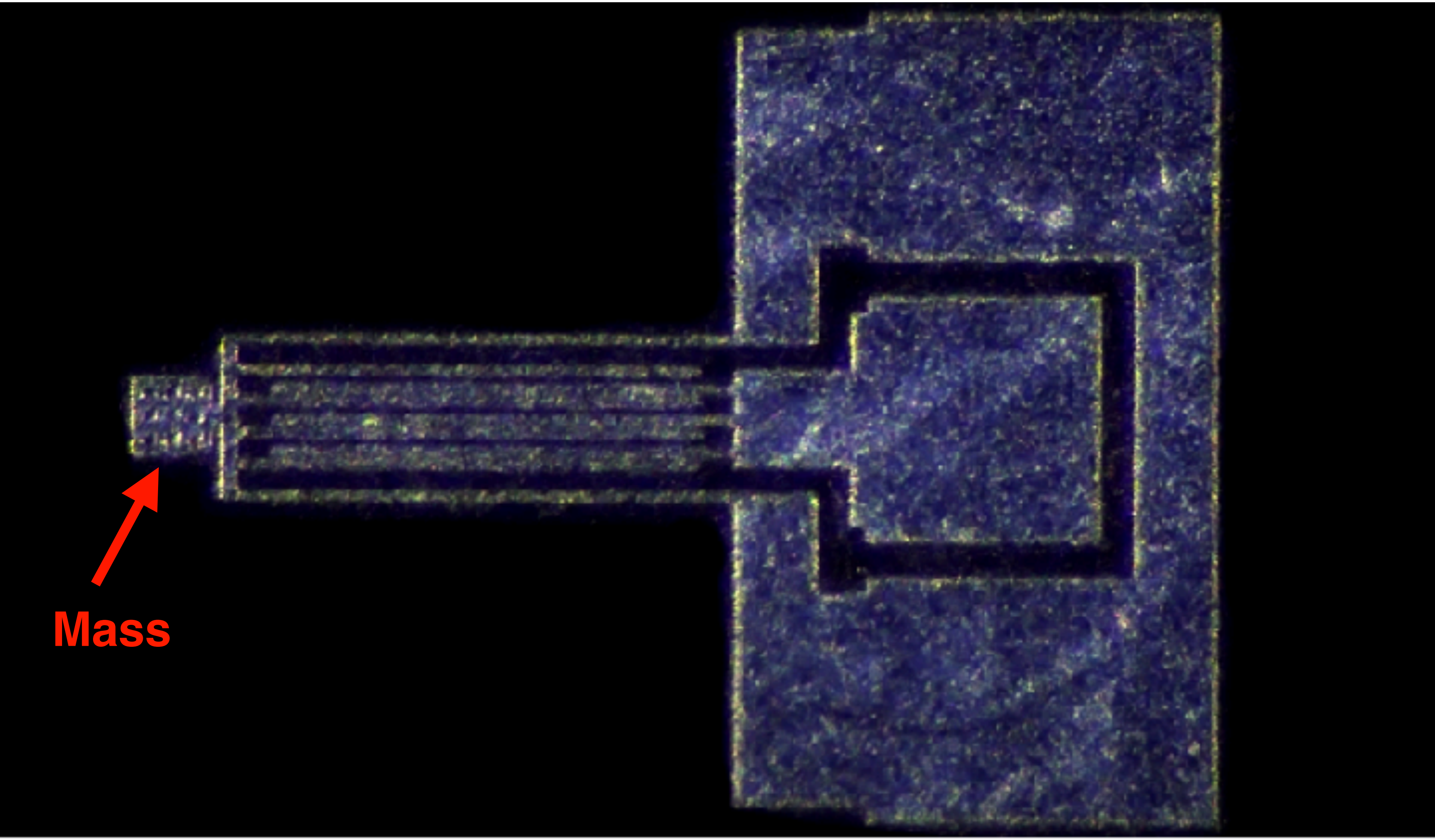
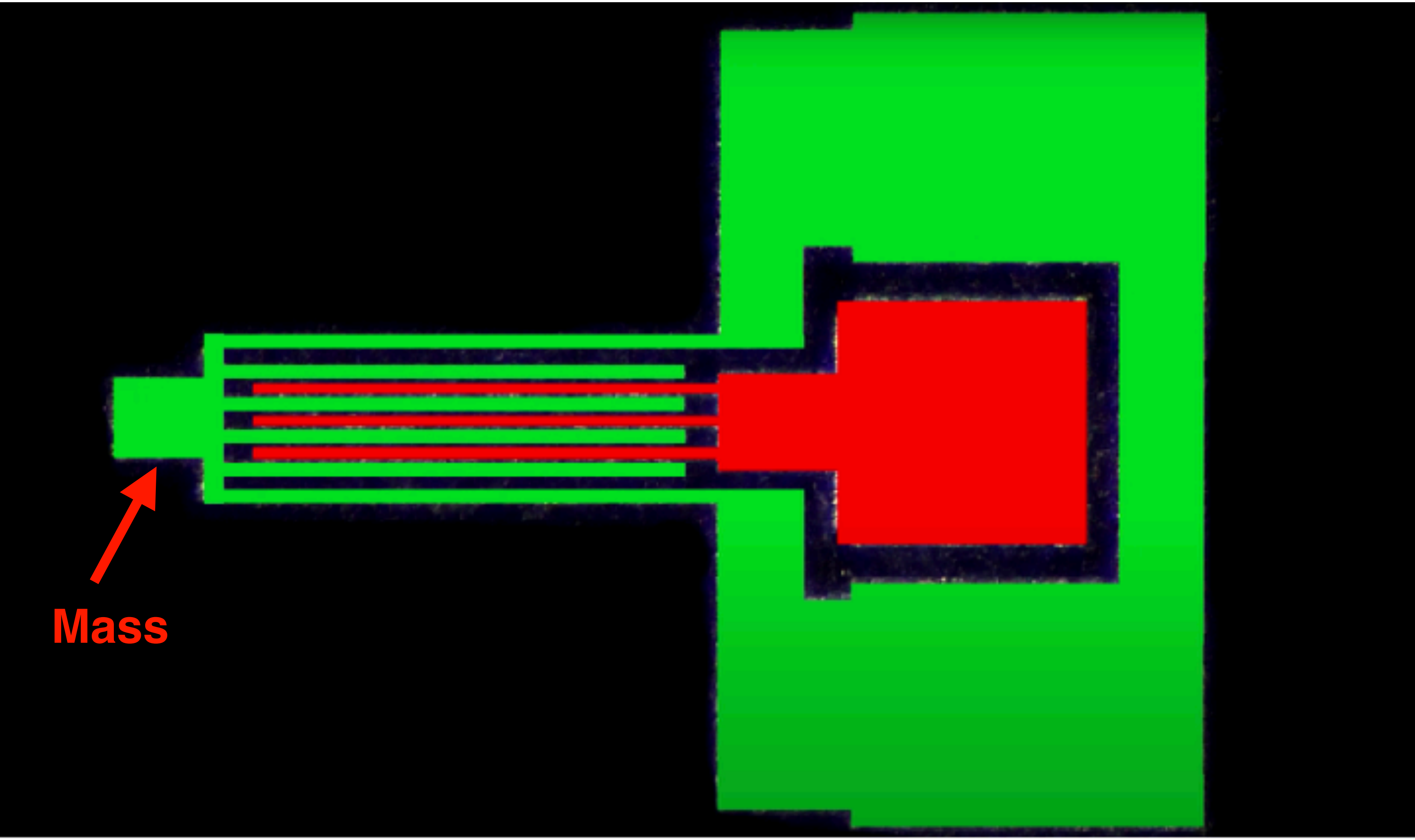
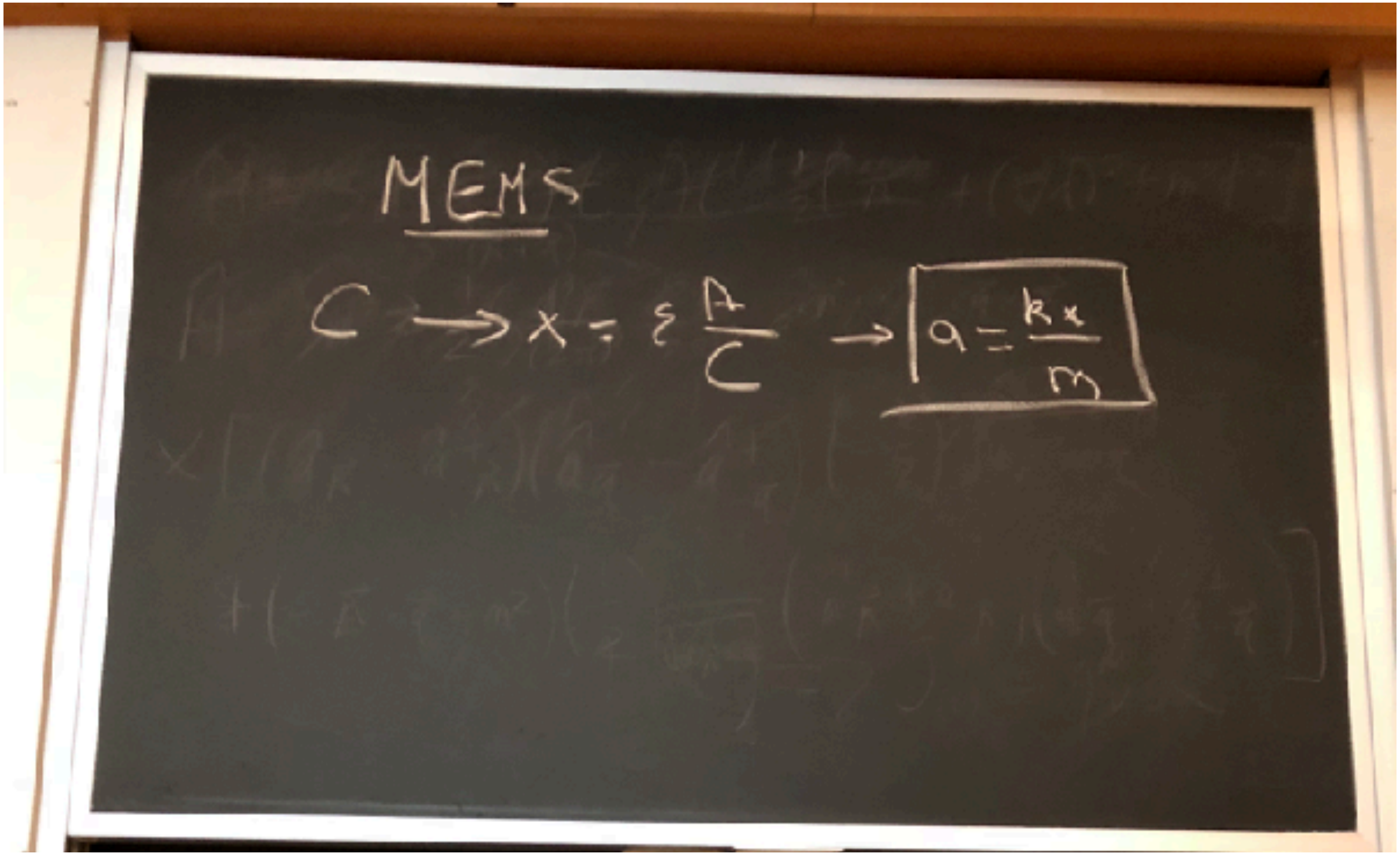


MEMS Accelerometer



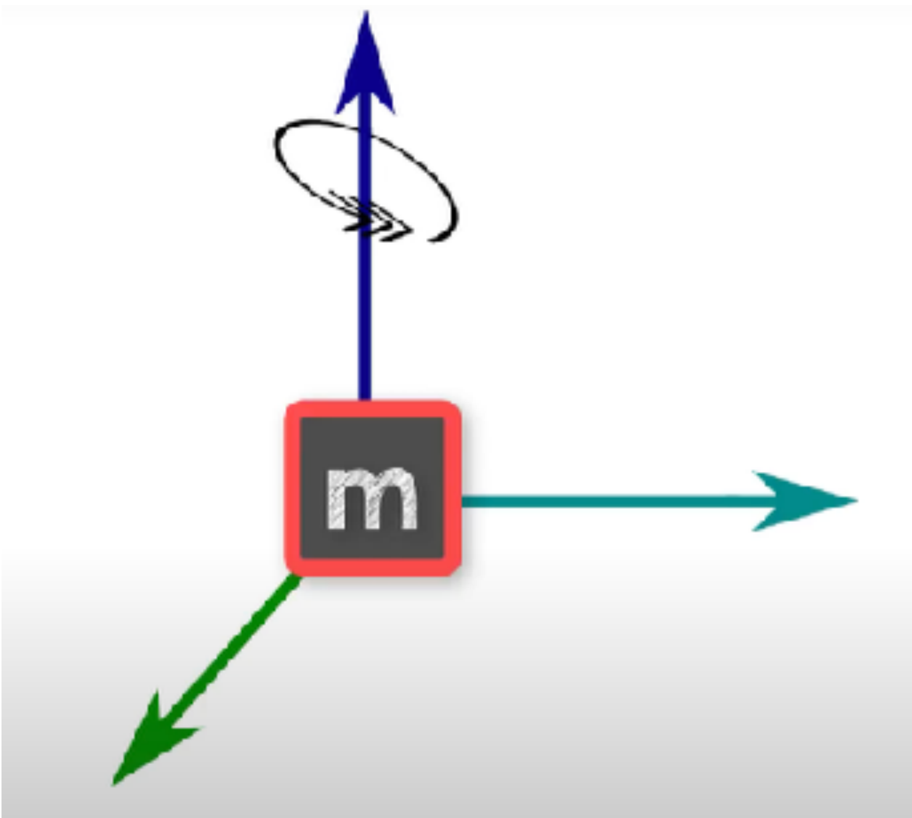
MEMS Accelerometer




MEMS
$C \longrightarrow x = \varepsilon \frac{A}{C} \rightarrow a = \frac{kx}{m}$

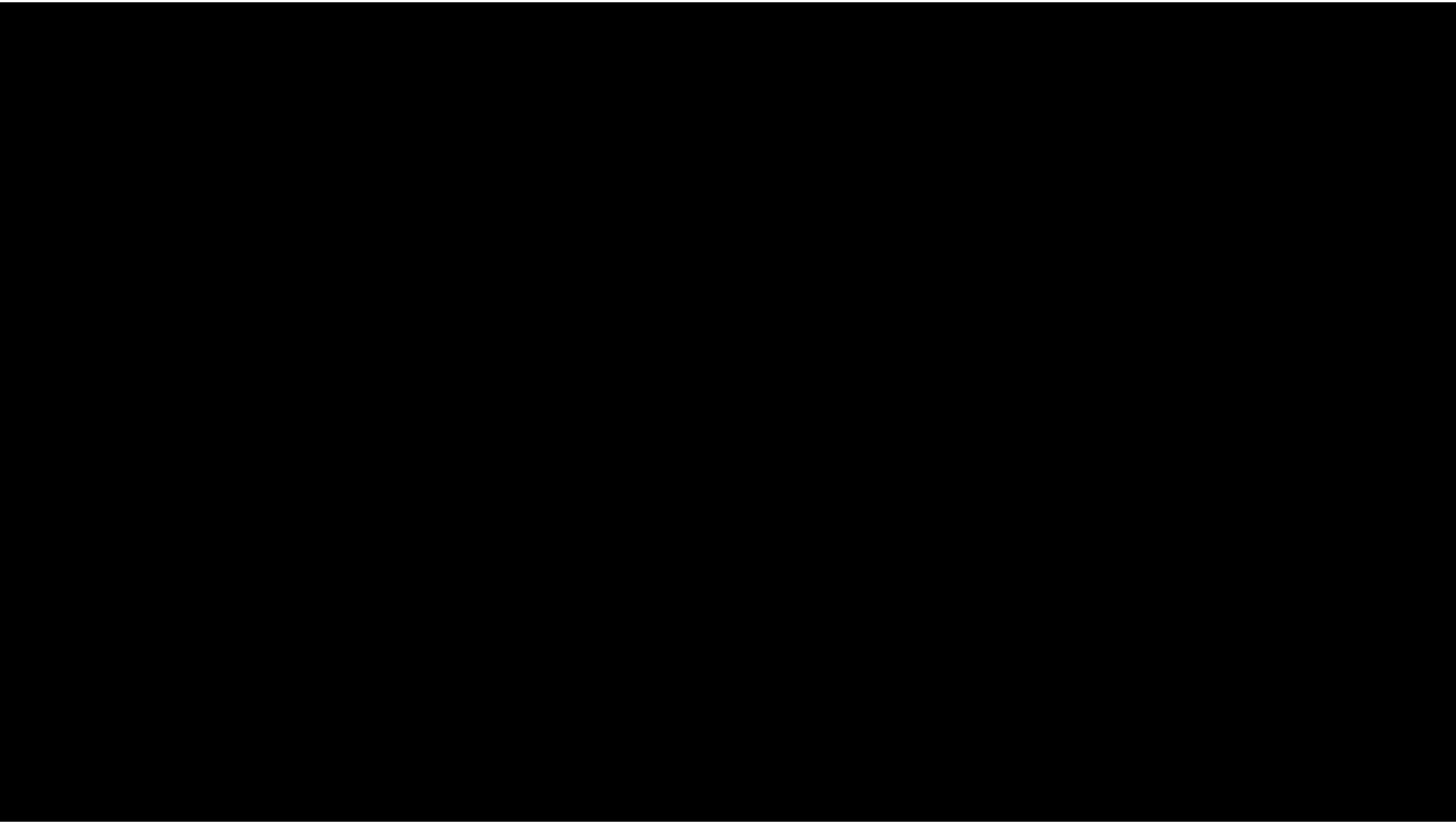
How Gyroscopes Work?

The Coriolis Effect



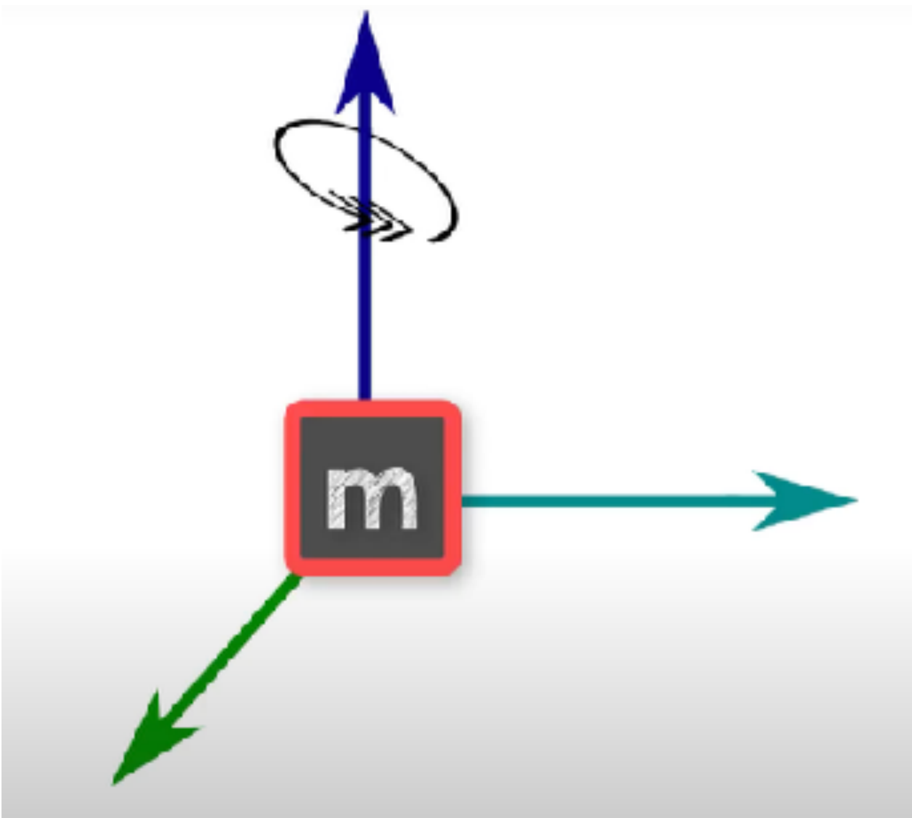
- Assume Vx
- Apply **ω**
- Experiences a fictitious force F(**ω,** Vx) following right hand rule

The Coriolis Effect



How Gyroscopes Work?

The Coriolis Effect

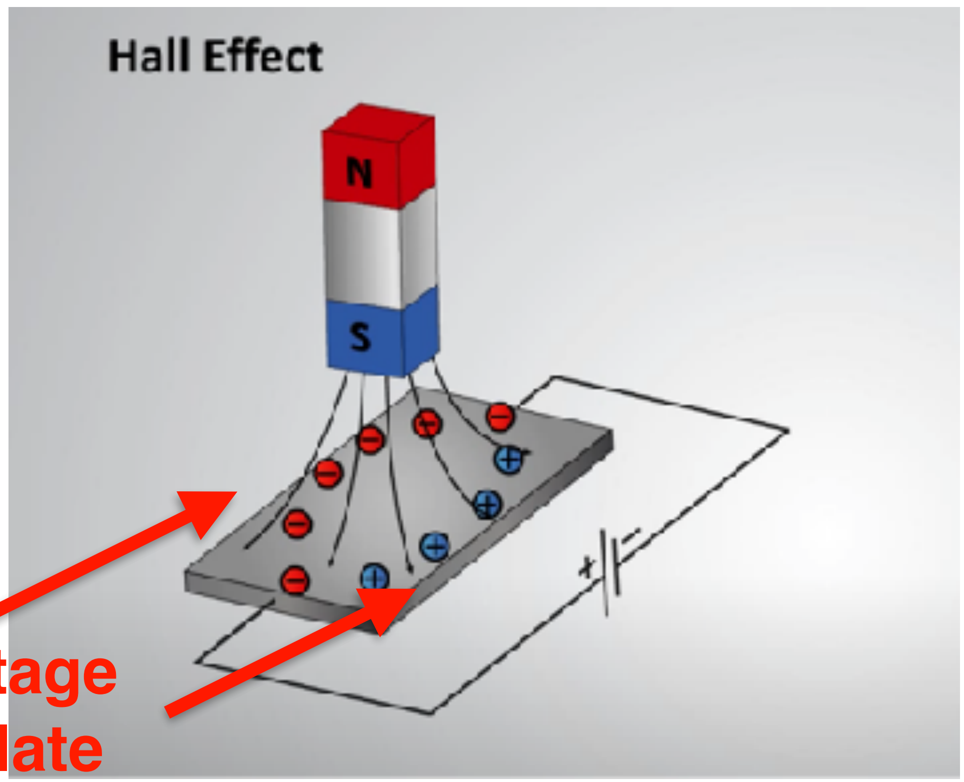


- Assume Vx
- Apply
- Experiences a fictitious force F(**ω,** Vx) following right hand rule

Can measure F in a similar fashion and use it to recover **ω**

How Magnetometers Work

- E.g., Compass
- Measure Earth's magnetic field



Measure voltage across the plate

Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- **Understanding Sources of Errors**
- Dead reckoning by fusing multiple sensors

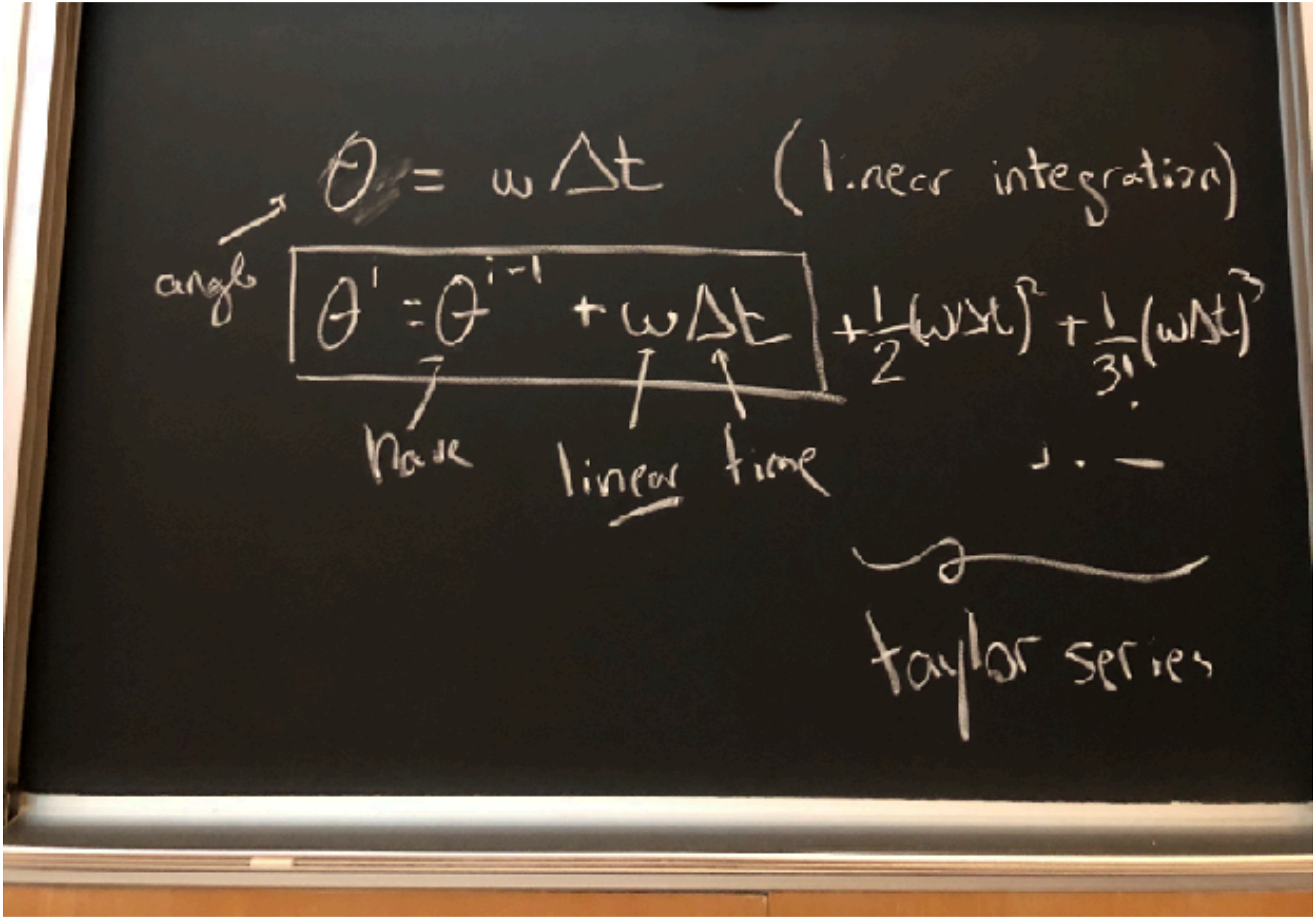
Gyroscope

Measured angular velocity:

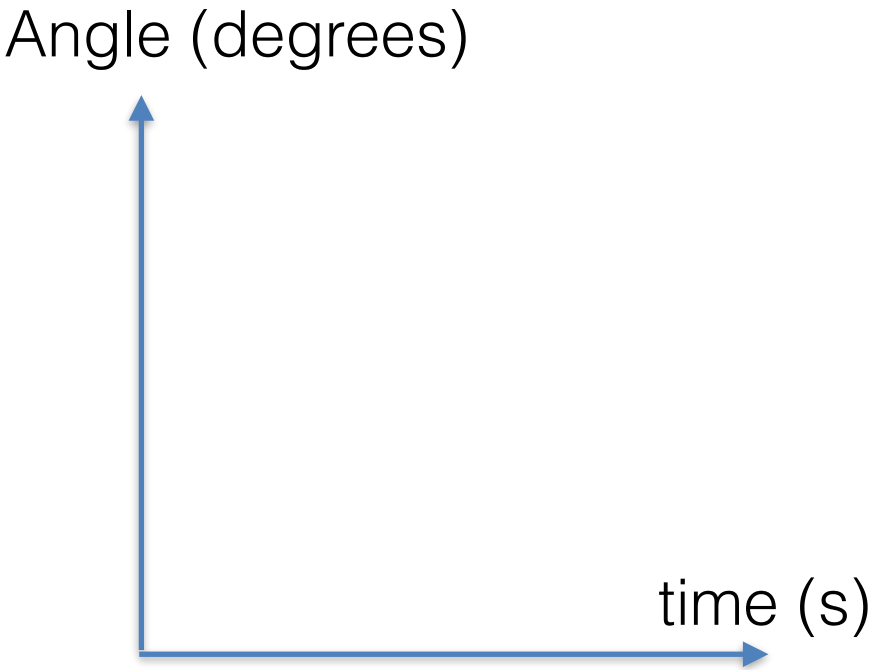
$$\widetilde{\omega} = \omega + b + \eta$$

- True angular velocity
- Bias
- Noise (Gaussian, zero mean)

- How to get from angular velocity to angle?
  - Integrate, knowing initial position


$\theta = \omega \Delta t$ (linear integration)
angle
$\theta^i = \theta^{i-1} + \omega \Delta t + \frac{1}{2}(\omega \Delta t)^2 + \frac{1}{3!}(\omega \Delta t)^3$
noise
linear
time
taylor series

Gyro Integration

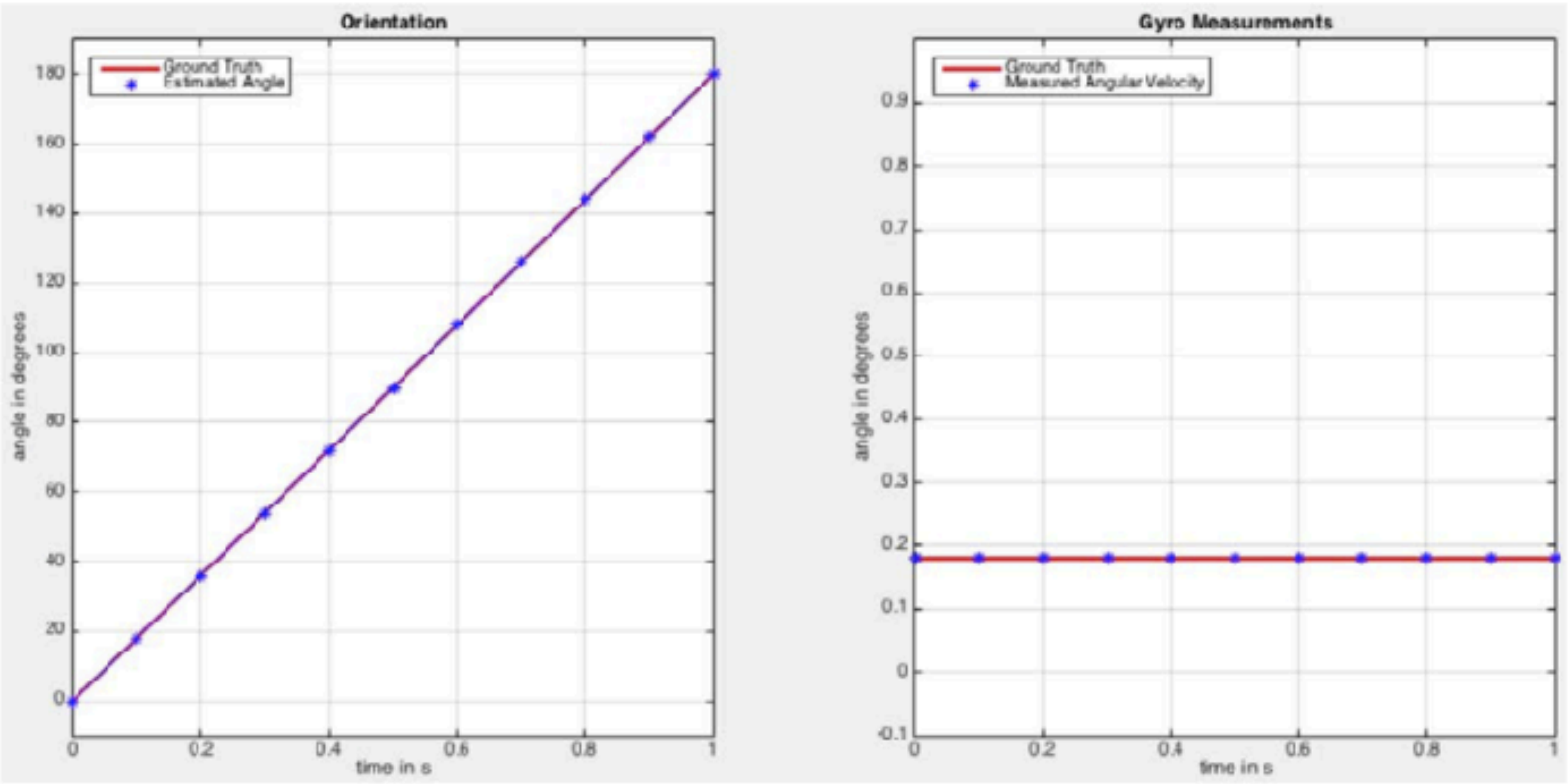


- Let's plot this for gyro measurement and for orientation
- Let's include ground truth and measured data for each

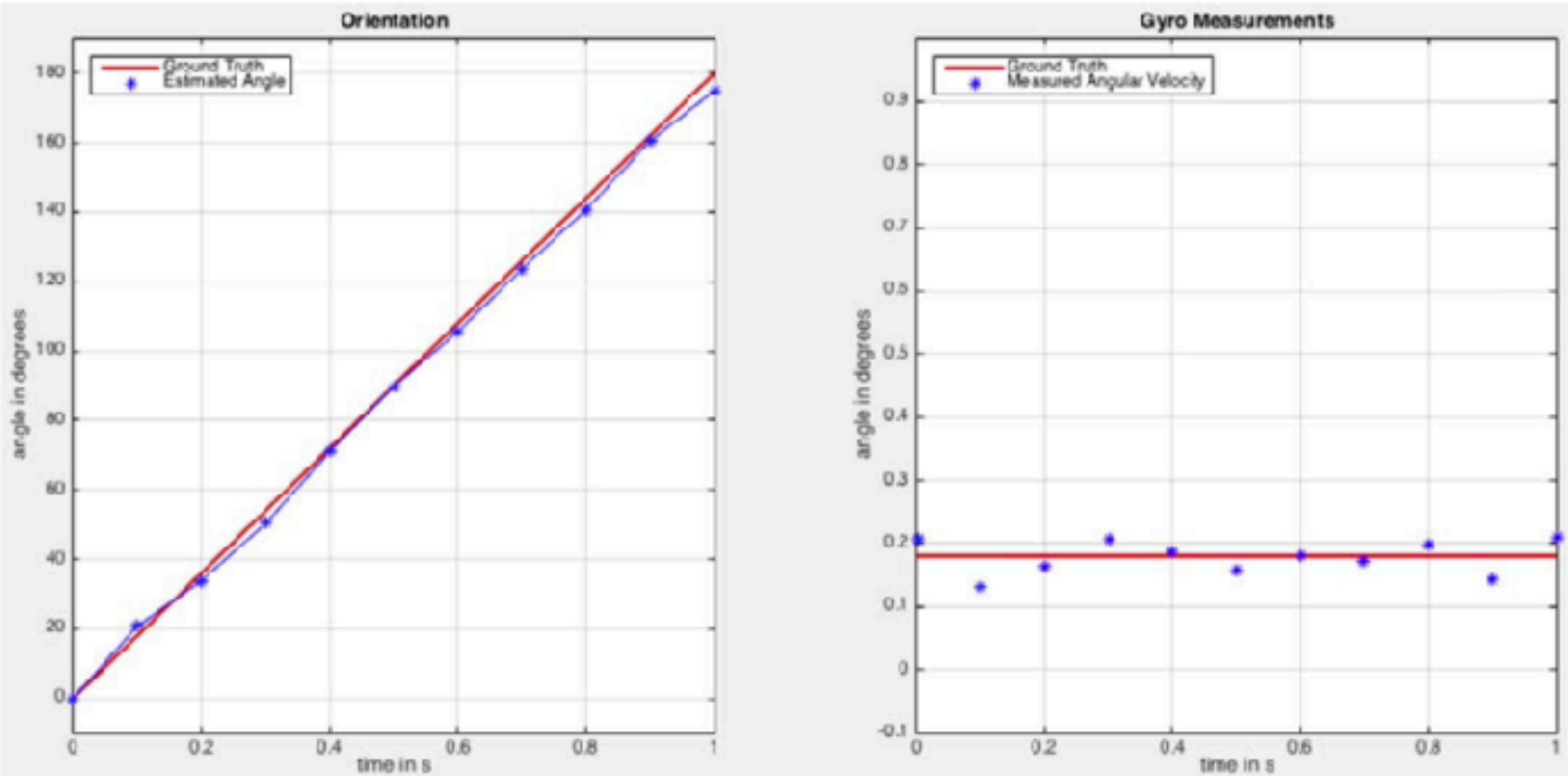
Consider:

- linear (angular) motion, no noise, no bias
- linear (angular) motion, with noise, no bias
- linear (angular) motion, no noise, bias
- nonlinear motion, no noise, no bias

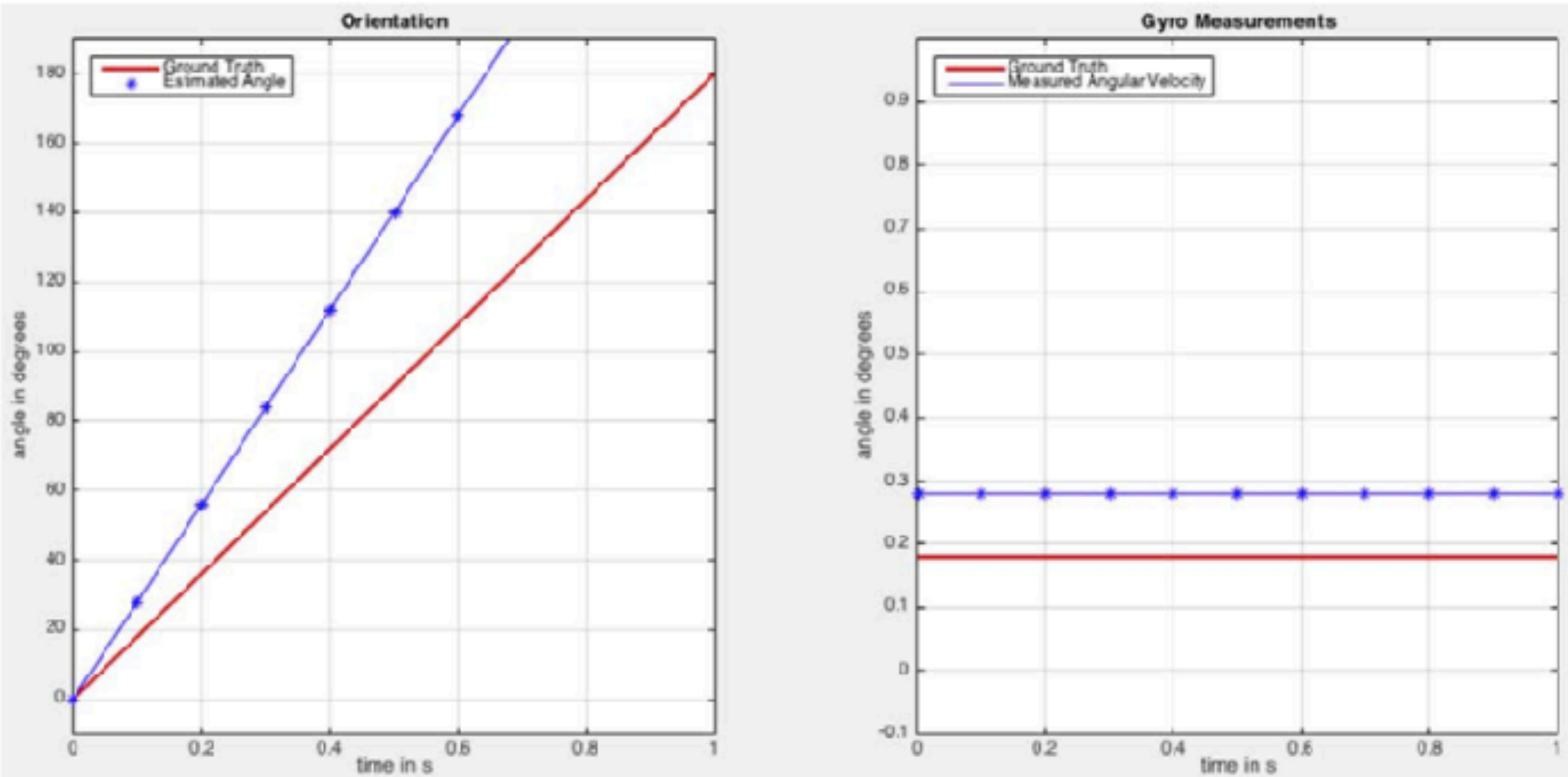
Gyro Integration: linear motion, no noise, no bias



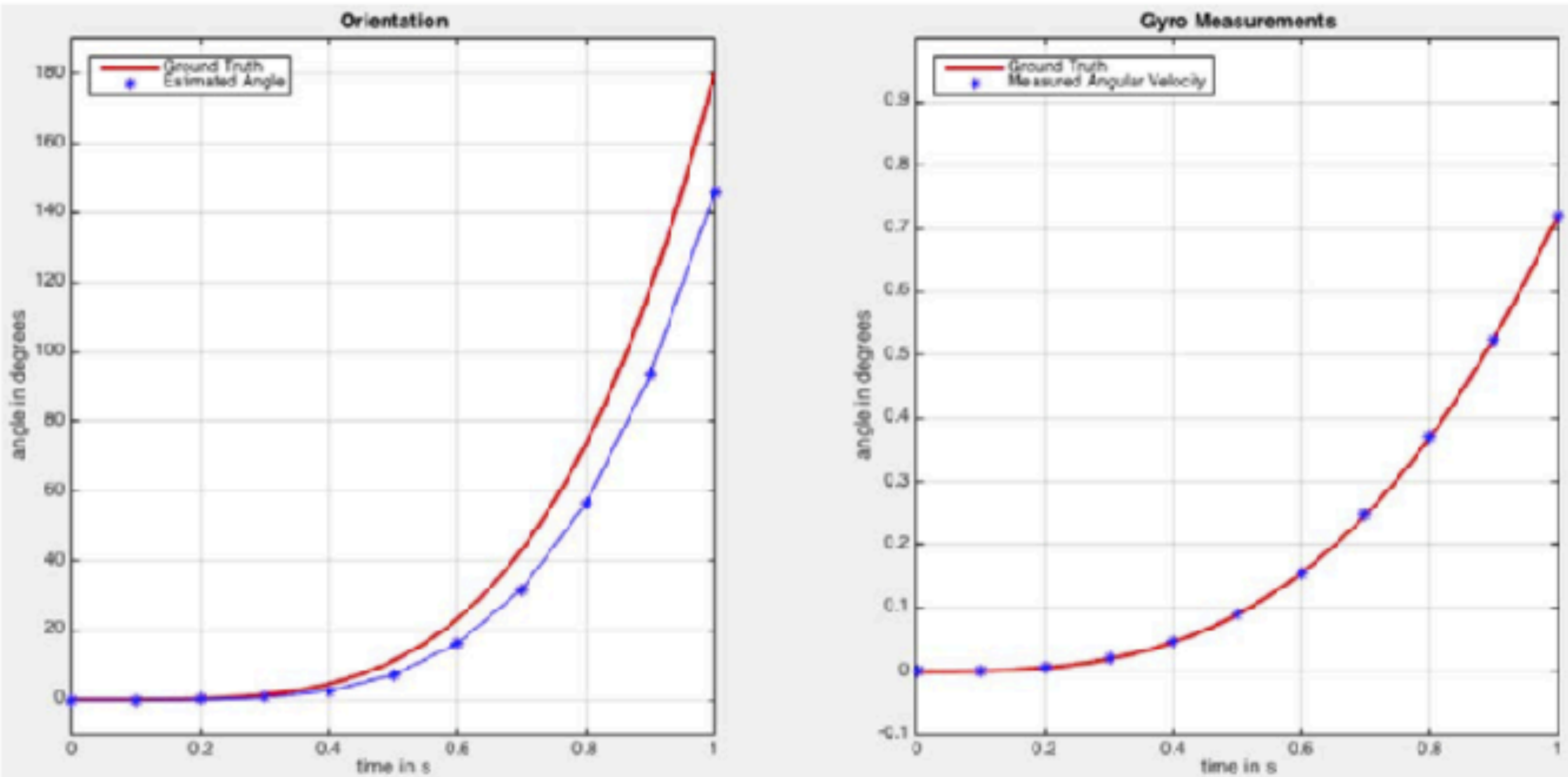
Gyro Integration: linear motion, noise, no bias



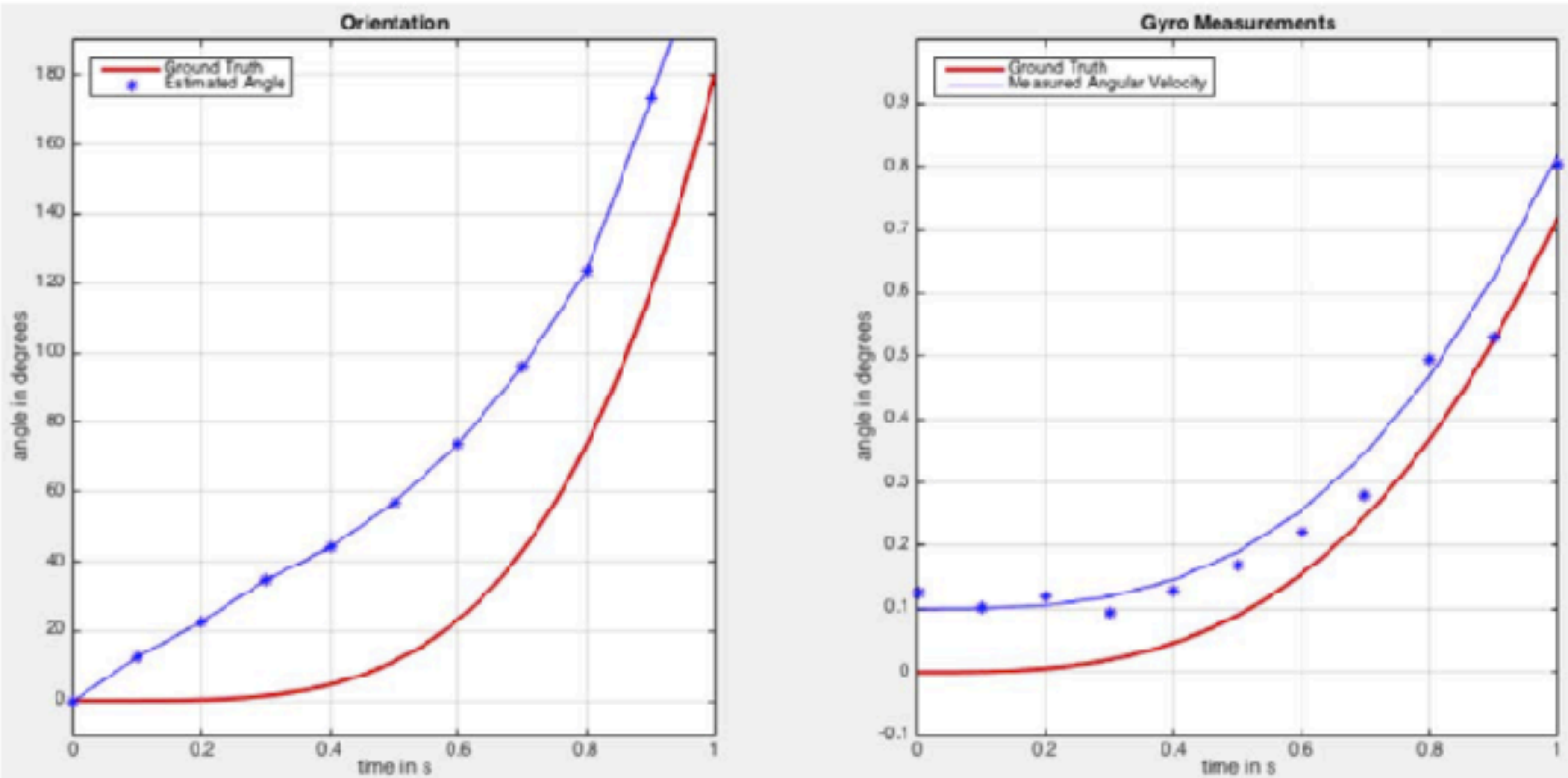
Gyro Integration: linear motion, no noise, bias



Gyro Integration: nonlinear motion, no noise, no bias



Gyro Integration: nonlinear motion, noise, bias



Gyro Integration aka *Dead Reckoning*

- works well for linear motion, no noise, no bias = unrealistic
- even if bias is known and noise is zero -> drift (from integration)
- bias and noise variance can be estimated, there sensor measurements used to correct for drip (sensor fusion)
- accurate in short term, but not reliable in long term due to drift

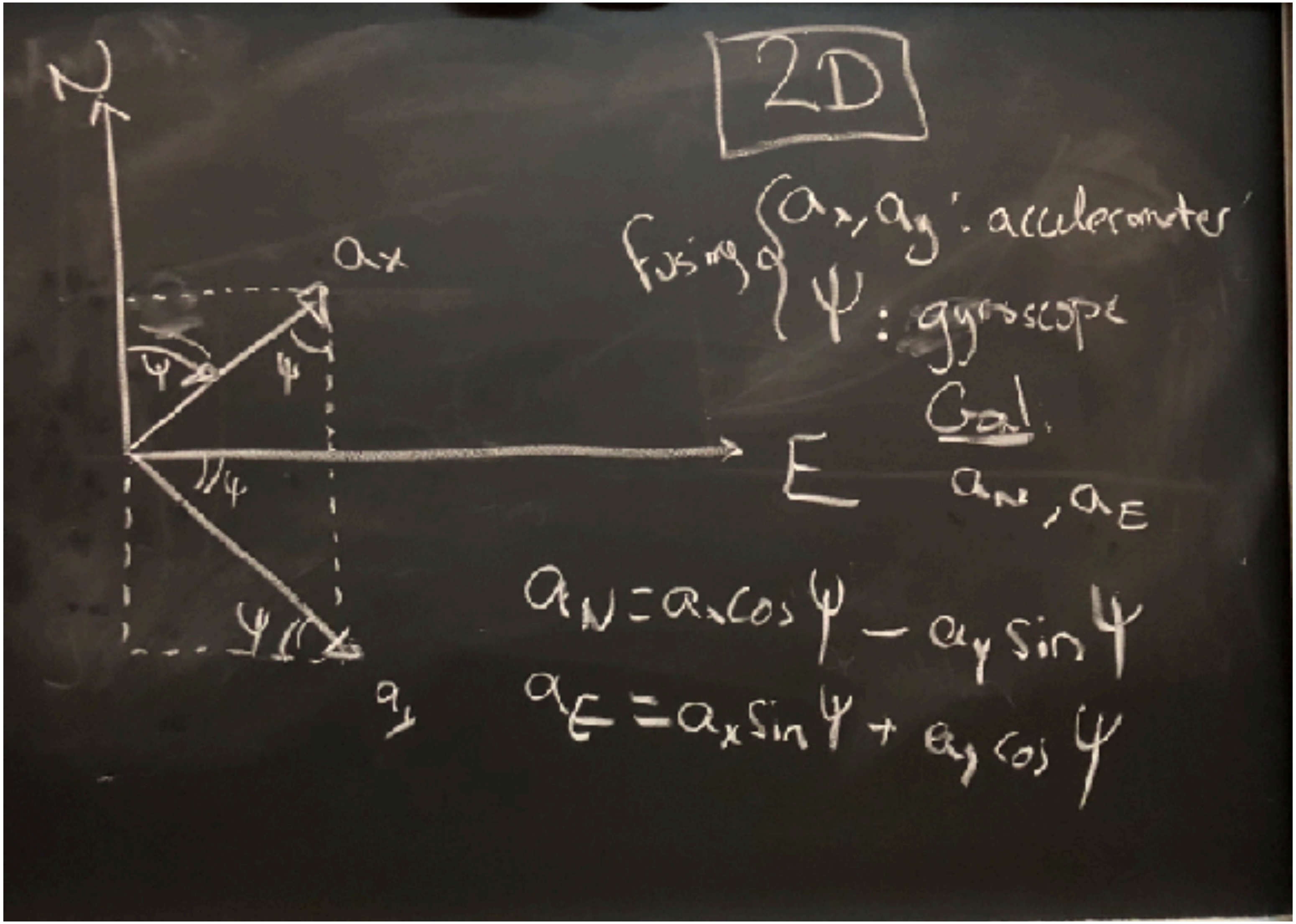
Rest of this Lecture

- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- Understanding Sources of Errors
- **Dead reckoning by fusing multiple sensors**

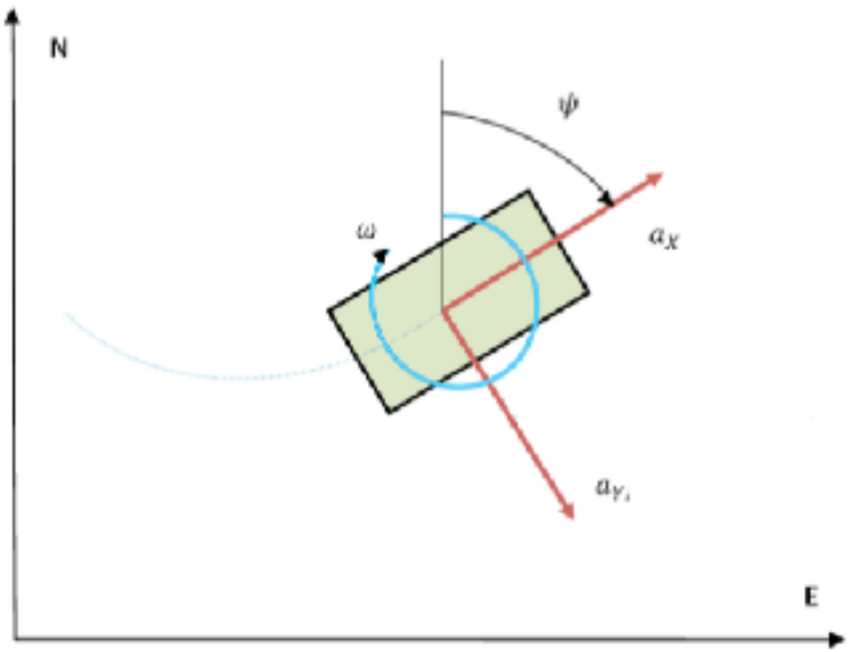
Dead Reckoning

- The process of calculating one's current position by using a previously determined position, and advancing that position based upon known or estimated speeds over elapsed time and course

- Key things to keep in mind:
    - Frames of reference
    - Orientation change


2D
fusing
a_x, a_y : accelerometer
ψ : gyroscope
Goal:
a_N, a_E
a_N = a_x cos ψ − a_y sin ψ
a_E = a_x sin ψ + a_y cos ψ
N
E
a_x
a_y
ψ

2D Inertial Navigation in Strapdown System



$$\begin{bmatrix} a_N \\ a_E \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} a_X \\ a_Y \end{bmatrix}$$

Source: Basic Principles of Inertial Navigation
Seminar on inertial navigation systems
Tampere University of Technology

2D Inertial Navigation in Strapdown System

$$\begin{bmatrix} a_N \\ a_E \end{bmatrix} = \begin{bmatrix} cos\psi & -sin\psi \\ sin\psi & cos\psi \end{bmatrix} \begin{bmatrix} a_X \\ a_Y \end{bmatrix}$$

$$V_N(t) = V_N(t_0) + \int_{t_0}^{t} a_N(t)dt \qquad X_N(t) = X_N(t_0) + \int_{t_0}^{t} V_N(t)dt$$

$$V_E(t) = V_E(t_0) + \int_{t_0}^{t} a_E(t)dt \qquad X_E(t) = X_E(t_0) + \int_{t_0}^{t} V_E(t)dt$$

ArmTrak (Tracking from Smart Watch)

Also fuse over time through hidden markov models (HMM)

Lecture Recap

- Importance of IMUs for navigation and sensing
- Coordinate systems and 6DOF
- IMU history and current use case cases
- Basic principles of operation of different IMU sensors: accelerometer, gyroscope, magnetometer
- Understanding Sources of Errors
- Dead reckoning by fusing multiple sensors